复旦大学数学科学学院

2012～2013学年第二学期期末考试

■ 高数A（下）A 卷参考答案

1. (1)$z_x(1,1)=3,\ z_{xy}(1,1)=-4.$

(2)$\begin{cases}-x+2y+2z-3=0\\ 2x-3y+5z-4=0\end{cases}$ 或$\dfrac{x-1}{16}=\dfrac{y-1}{9}=\dfrac{z-1}{-1}.$

(3)$2\pi.$

(4)$\dfrac{4\pi}{3}.$

(5)$\dfrac{2\pi}{3}.$

(6)2小时。

2.$z'(s)=\dfrac{\sqrt{2}}{2}.$

3.因为$\lim_{x\to 0}\dfrac{f(x)-f(0)-f'(0)x}{x^2}=\dfrac{f''(0)}{2}$,

所以$\exists N_0>0\forall n>N_0$: $|f(\dfrac{1}{n})-f(0)-f'(0)\dfrac{1}{n}|<\dfrac{|f''(0)|+1}{n^2}$, 因而：

(1)$f(0)\neq 0$时，级数发散；

(2)$f(0)=0, f'(0)\neq 0$时，级数条件收敛；

(3)$f(0)=f'(0)=0$时，级数绝对收敛。

4.记$r=\sqrt{x^2+y^2}$, 则$z_x=2xf(r^2)+2xr^2f'(r^2)$,

$z_{xx}=2f(r^2)+[2r^2+8x^2]f'(r^2)+4x^2r^2f''(r^2)$,

同理：$z_{yy}=2f(r^2)+[2r^2+8y^2]f'(r^2)+4y^2r^2f''(r^2)$,

所以：$z_{xx}+z_{yy}=4f(r^2)+12r^2f'(r^2)+4r^4f''(r^2)=0.$

记$y(t)=f(e^t)$, 则：$y''+2y'+y=0,\ y=(C_1+C_2t)e^{-t}.$

所以：$f(x)=\dfrac{C_1+C_2\ln x}{x}$, 代入初始条件得：$f(x)=\dfrac{\ln x}{x}.$

5.记$r=\sqrt{x^2+4y^2+4z^2}$, 则:

$\dfrac{\partial}{\partial x}(\dfrac{x}{r^3})=\dfrac{1}{r^3}-3\dfrac{x^2}{r^5},\ \dfrac{\partial}{\partial y}(\dfrac{y}{r^3})=\dfrac{1}{r^3}-3\dfrac{4y^2}{r^5},\ \dfrac{\partial}{\partial z}(\dfrac{z}{r^3})=\dfrac{1}{r^3}-3\dfrac{4z^2}{r^5}.$

所以：$\dfrac{\partial}{\partial x}(\dfrac{x}{r^3})+\dfrac{\partial}{\partial y}(\dfrac{y}{r^3})+\dfrac{\partial}{\partial z}(\dfrac{z}{r^3})=0.$

原式=$\displaystyle\iint\limits_{x^2+4y^4+4z^2=1\text{的外侧}} x\mathrm{d}y\mathrm{d}z+y\mathrm{d}z\mathrm{d}x+z\mathrm{d}x\mathrm{d}y=3\iiint\limits_{x^2+4y^4+4z^2\le 1}\mathrm{d}x\mathrm{d}y\mathrm{d}z=\pi.$

6(1)$a_0=1, a_n=0(n\geq 1),\ b_n=\dfrac{1+(-1)^{n-1}}{n\pi}$,

$$f(x) \sim S(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{(2n-1)\pi} \sin(2n-1)x$$

$S(\frac{7\pi}{2}) = S(-\frac{\pi}{2}) = 0,\ S(7\pi) = S(\pi) = \frac{1}{2}.$

(2)

$$I = \int_{-\pi}^{\pi} \left[\left(f(x) - g(x) \right)^2 + g^2(x) \right] \mathrm{d}x = \int_{-\pi}^{\pi} \left[2\left(\frac{f(x)}{2} - g(x) \right)^2 + \frac{1}{2} f^2(x) \right] \mathrm{d}x$$

所以$A_i = \frac{a_i}{2},\ i = 0, 1, \ldots, 10,\ B_i = \frac{b_i}{2},\ i = 1, \ldots, 10.$

又解：由$\frac{\partial}{\partial A_i} I = 0$得：$A_i = \frac{a_i}{2},\ i = 0, 1, \ldots, 10,$

由$\frac{\partial}{\partial B_i} I = 0$得：$B_i = \frac{b_i}{2},\ i = 1, \ldots, 10.$

7(1)由$\frac{\partial}{\partial y} P = \frac{\partial}{\partial x} Q$解得：$a = 1, \lambda = 1.$

但是，此时沿单位圆$x^2 + y^2 = 1$逆时针的积分$= 2\pi,$

所以无论a, λ如何取值，都不能使积分完全与路径无关。

(2)由$\frac{\partial}{\partial y} P = \frac{\partial}{\partial x} Q$解得：$a = 4, \lambda = 2.$

此时$P\mathrm{d}x + Q\mathrm{d}y = \mathrm{d}\frac{-2y^2}{x^2 + y^2}$, 因而积分与路径无关。